Abstract—In order to overcome the shortcomings of the shape from shading (SFS) algorithm in items of clearness and smoothness of silhouette in reconstructed objects, a modified SFS algorithm is proposed in this paper. Firstly, we determine the primary factors affecting reconstruction by analyzing the Lambertian light reflection model. Secondly, we perform image restoration and background smoothing by extracting object silhouette using edge detection to separate the object and background. Finally, the object is reconstructed using the SFS algorithm. Theory and simulation results show that, compared with the traditional algorithms, the performance of the modified algorithm was efficiently improved, which can improve the accuracy of the reconstructed shape and the continuity of the silhouette of reconstructed objects by reducing the reconstruction error effectively.

I. INTRODUCTION

3D shape recovery of objects has been a central problem in computer vision, which task is to model the surface three-dimensional information from a single 2D image or multiple images. The idea of shape-from-shading (SFS) [1] is reconstructing the height of object surface by applying the shading information from 2D image, which is a challenging task requiring several processes such as illumination parameter estimation and depth calculation. It is well known that this issue is ill-posed, and many questions in this field are still open, such as the uniqueness of solutions. A large number of papers which study on SFS present different methods to solve this problem in effect, and take the advantages of new digital image processing techniques, it is possible to optimize the SFS algorithm to improve the recovery shapes.

3D reconstruction combines the theories and approaches in the field of computer graphic, image processing and computer vision. Recently, 3D reconstruction based on images has been widely applied on industrial technology, medical imaging and the science and technology for national defense, etc.

In classical algorithms, it usually assumes that the surface of the object is smooth and the reflection model is Lambertian illumination model, which refrains the brightness of the surface. After that, by solving simultaneous equations, the height information can be gained. However, most of the 2D image is not calibrated and it faces many difficulties.

In this paper, we are going to address the problem of lacking of clearness and smoothness of silhouette in recovery shapes by using image restoration and background smoothing. We assume that the reflectance map conforms the Lambertian model. The contributions in this paper are as follows:

1) To improve the clearness and smoothness, we analyze the dominant factors which play an important role in 3D reconstruction. By analysis, we can determine to solve the problem adding constraints to these factors. In consequence, we can get a better recovery of images in real world.
2) Edge detection is employed to separate the object and background in order to redefine the grey value of background, which can reduce the computation in reconstruction effectively. By image inpainting, we can get more accurate information in 2D image, which is beneficial to give rise to the smoothness in recovery shapes.

There are main two serious defects in most of the existing methods: 1) edge profile is not clearly 2) there is large error of profile of reconstruction. Based on the two preferable properties, we have proposed a new method based on shape-from-shading to reconstruct object from a single image. As a result, this approach achieves better result in terms of clearness and smoothness.

The remainder of this paper is organized as follows. Section II gives a brief review of related work. In section III, we describe the Lambertian illumination model and the theory of SFS. We will present the algorithm based on silhouette optimization in IV. In section V we present our experimental results and analysis. Finally, conclusions of this paper are drawn in section VI.

II. RELATED WORKS

In this section, we introduce some valuable work and existing problems in 3D reconstruction from a single image in SFS method.

Generally, SFS can be implemented by different approaches. Based on the ideas and models of traditional methods, many researchers have improved these algorithms, which can be divided into four classes: methods of global minimum [2], methods of maximal parsimonious [3], methods using local analysis [4] and linearization technique...
Tankus et al. [6] proposed an algorithm to reconstruct object under perspective projection based on perspective fast marching method. But in this method, the initial data will affect the definition of objects. Based on SFS, an approach of obtaining surface depth information by solving Hamilton-Jacobi (HJ) equation under perspective projection has been proposed [7]-[10]. However, the problem of occluded boundary is not resolved in some situations. Wang et al. [11] uses high-order local Lax-Friedrichs (LLF) flux splitting scheme to increase the accuracy of partial derivatives.

SFS problems also can be solved by applying global propagation, which mainly begins the reconstruction from a singular point [12], [13], [14]. For global based approaches, it is usually need an initial assumption. Moreover, they also need a great deal of computation but the convergence is not always effective. Pradoss et al. [15] proposed a unifying and rigorous SFS method based on the notion of continuous and discontinuous viscosity solution. They pointed out that the method is robust to pixel noise. Zhong et al. [16] also proposed another global based approach to SFS.

In other papers, some researchers have attempted to enhance the applicability of SFS approaches by simulating the physics of it in a realistic mode. Lee et al. [17] and Prodas et al. [18] take into account the non-Lambertian situation. The methods often contain two steps: the first one is computing the weighted distance functions from all singular points and the second one is the merging of the surfaces.

### III. SFS AND LAMBERTIAN ILLUMINATION MODEL

#### A. Lambertian illumination model

We start by giving a brief outline of the SFS problem and introducing the basic assumptions. The brightness of an image is influenced by several factors, such as light source, the material of the object, the distance between light source and object and so on. To simply the problem, we introduce Lambertian illumination model, which satisfies the following assumptions:

1) The scene is illuminated by a single point light source which is at infinity, or the light source is uniform light;
2) The surface of the body appears equally bright from all directions and it will reflect the incident light completely.

According to the relation between reflection characteristic of the curved surface, the distribution character of light source and albedo of the surface, in equation (1) is the “image irradiance equation:

$$ R(p, q) = I_p \rho \cos \theta = I_s \rho \cos \angle(n, s) $$

Where $I_p$ is intensity of incident light, and $\rho$ is the reflectance coefficient, which is different under different materials. $\theta$ is the incident angle. $s$ represents the direction of light source and $n$ is the unit normal to the surface at point $(x, y)$. $R(p, q)$ is the reflectance function, containing the value of the light re- emitted by the surface at point $(x, y)$ in the direction of $p$ and $q$, and $p$ and $q$ are the slope of the surface along $x$ and $y$ direction, respectively.

$$ I(x, y) = R(p(x, y), q(x, y)) $$

Where $I(x, y)$ is the brightness measured in the image at point $(x, y)$. Combining Eq. (2) with Eq. (3), we can rewrite $E(x, y)$ by:

$$ I(x, y) = \rho \frac{pp_i + qq_j + 1}{\sqrt{p^2 + q^2} + 1} $$

There are two unknown quantities in this equation. However, we can only get to know the information of brightness from the image, so this problem is ill-posed. In order to solve these problems, other restraint conditions should be introduced. As we all known that, the brightness information and silhouette are the main factors affecting reconstruction.

### IV. SFS ALGORITHM BASED ON SILHOUETTE OPTIMIZATION

In the edge of an image, there is much information about the local property, which represents the discontinuity, that is, the areas of greylevel vary remarkably. Since the information, contained in the edge areas, provides the necessary condition that needed in the distinguishing of target object and background, it is important to use edge detection and then
extract the silhouette feature. Considering the features, we can optimize the reconstruction of our object. The processing flow is showed in Fig.2.

![Fig. 2. 3D Reconstruction Algorithm flow chart](image)

### A. Silhouette extraction of 2D information

In this paper, we apply Canny [19] operator as the gradient operator of edge detection. Before silhouette extraction, image smooth process is proposed, in which using Gaussian Filter. Secondly, calculating the gradient by computing the first partial derivative of Gaussian function, so it is easy to detect the local maximum of grads module. Then the weak edge \(E_1\) can be gained by low threshold value while strong edge \(E_2\) can also be gained by high threshold, where \(1 \subseteq E_1 \subseteq E_2\). Finally, outputting edge \(E\), where \(E\) is the connected component linked to \(E_1\) and \(E_2\).

### B. Background separation and image inpainting

After edge extraction, we consider to separate the target object and background. At first, in order to calculate the mean value of gray \(E_{avg}\)(without inside and outside of silhouette), making a statistics to the gray value of the contour line. And then, separate the area of background and the area of target object; for target object, stating the same; processing the background and assign the new gray value \(E_{avg}\) to it. \(E_{avg}\) is computed by following equation:

$$E_{new}(x, y) = \begin{cases} E_{avg}(x, y) & (x, y) \in E_{background} \\ E_{old}(x, y) & (x, y) \in E_{substance} \end{cases}$$

Where \(E_{background}\) is the gray value of background and \(E_{substance}\) is the gray value of target object. From above process, the whole background will has the same value of gray, so that it can get the ideal effect of reconstruction. In addition, since the gray value of the background is similar to the value of external contour, the problem of existing of steep slope can be solved effectively.

In practice, the gradient of the surface may be large and gray level changes greatly in these areas, so that it may occur defects, such as pit, burr, etc. In this paper, we apply total variational image restoration method [8] to perform image smooth process, which can be effectively restrained above problems. Total variational restoration model expressed by:

$$\frac{\partial u}{\partial t} = \nabla \left( \frac{\nabla u}{\| \nabla u \|} \right) + \lambda\varepsilon (u - u_0)$$

Where \(u\) is the pixel under repair, \(u_0\) is the known pixel in the boundary of the damaged region. The pixel in unknown area can be repaired according to Eq. (6).

### C. 3D image reconstruction

The information about 3D appearance can be obtained by reflection equation. According to Eq. (4), using finite difference to compute \(p\) and \(q\) as following:

$$p = \frac{\partial z}{\partial x} = z(x+1,y) - z(x,y) - z(x-1,y)$$

$$q = \frac{\partial z}{\partial x} = z(x,y+1) - z(x,y) - z(x,y-1)$$

A three-dimensional object can be represented by a surface equation \(z = f(x, y)\), where \((x, y)\) is a point in a three-dimensional coordinate system \((xyz)\). And then liberalizing reflectance function \(R(p, q)\). Finally, for any \((x, y)\), according to \(E(x, y)\), using Taylor series expansion can get following results

$$f(z(x,y)) = f(z^{(n-1)}(x,y)) + (z(x,y) - z^{(n-1)}(x,y)) \frac{d}{dz(x,y)}\left(z^{(n-1)}(x,y)\right) = 0$$

$$z^n(x,y) = z^{(n-1)}(x,y) + \frac{-f(z^{(n-1)}(x,y))}{\frac{d}{dz(x,y)}f(z^{(n-1)}(x,y))}$$

Where

$$\frac{df(z^{(n-1)}(x,y))}{dz(x,y)} = \frac{p_1 + q_1}{\sqrt{p_1^2 + q_1^2} + 1 + \sqrt{p_1^2 + q_1^2} + 1}$$

Given initial value \(z^0(x,y) = 0\), then the depth of surface can be obtained by iterative computations.

### V. EXPERIMENTAL RESULTS AND ANALYSIS

In order to evaluate the performance of our algorithm, we perform the experiments. The two images in our simulations are an image with pixel 128×128 of vase and an image with pixel 256×256 of flower.

Experiment 1: The classical vase image in SFS is used in this experiment, and the image size is 128×128 pixels. Fig.3 and Figure.4 shows the original image and two dimensional contour respectively. Fig.5(a) shows the result of reconstruction of traditional SFS algorithm, while Fig.5(b) is the result of our algorithm.

In the experiment, we apply mesh function in Matlab to draw the surface of 3D object. From Fig.(5), we can see that, reconstruction result in (b) has more smooth surface and
better edge compared with traditional algorithm. In table I, we compared the error of height between the two algorithms.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Error of height</th>
<th>Time/s</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>average</td>
<td>maximum</td>
</tr>
<tr>
<td>Our Algorithm</td>
<td>8.13</td>
<td>24.85</td>
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Experiment 2: Using flower image with complex edge, and the size is 256×256 pixel. Fig.8 (a) shows the result of reconstruction of traditional SFS algorithm, while Fig.8(b) is the result of our algorithm. Comparing the result in Fig.8(a) and (b), we can draw the conclusion that our algorithm can get more smooth edge.
VI. CONCLUSION

Considering the existing problems in traditional SFS algorithm, applying edge detection and other image processing methods, we present a 3D reconstruction algorithm from a single 2D image object based on silhouette optimization. The simulation results show that, compared with traditional algorithms, our algorithm can effectively distinguish the target and background and improve the similarity, so the reconstruction object is much more clearly, it will have a wide field of application with good prospects.

REFERENCES