

# Magnetic–Gravity Gradient Inversion for Underwater Object Detection

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**Abstract.** A new underwater object detection method based on joint Gravity-Gradient and Magnetic-Gradient Inversion algorithms is proposed in this paper. Magnetic gradient and gravity gradient anomalies induced by an underwater object can be measured and inverted to estimate the relative changes in distance between underwater object and underwater vehicle. The weight least squares estimation is introduced to combine equations of magnetic gradient and gravity gradient inversion. Therefore, the joint inversion method gets an optimal relative position of underwater object using two kinds of data. Simulation results show that the proposed method is more efficient than the previous Joint gravity – gradient tensor inversion and magnetic gradient tensor inversion respectively.

**Keywords:** Magnetic gradient inversion · Gravity gradient inversion · Underwater object detection · Weight least squares estimation

## 1 Introduction

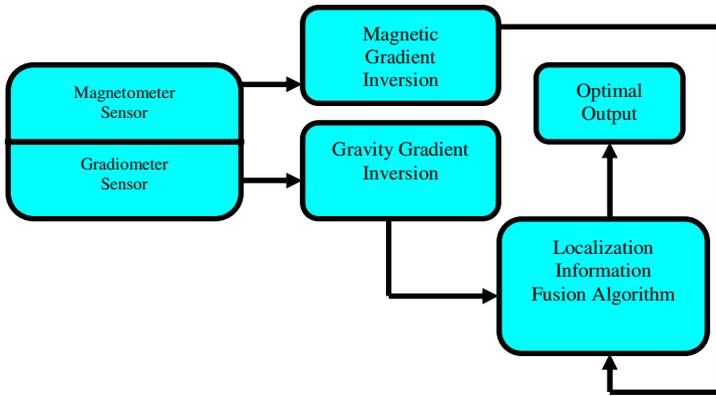
Underwater object detection is an area of research with broad commercial and military applications. How to detect underwater abnormal object is still a major challenge in underwater navigation field nowadays [1-2]. Over past decades, underwater object detection systems based on gravity gradient and magnetic anomaly have been researched widely [3-8]. In the field of underwater object detection based on gravity gradient tensor, Lin Wu used gravity gradient inversion method to detect abnormal objects in underwater environment [4]. Furthermore, in reference [5], joint gravity and gravity gradient inversion is presented for passive subsurface object detection. In reference [7], Zu Yan presented a novel method for underwater object detection based on the gravity gradient differential and the gravity gradient differential ratio caused by the relative motion between the AUV and the object. In the field of underwater object detection based on magnetic gradient tensor, YuHuang proposed a localization method combined with magnetic gradient tensor and draft depth [8]. Hao yan-ling put forward an available solution, where the geomagnetic anomaly is inverted a magnetic dipole target and the relative position of vehicle is determined by magnetic magnitude and gradients of targets [9].

After considering the invisibility requirement of AUVs and the existing problems in using gravity gradient and magnetic gradient for detecting objects, this paper proposed a novel and practical solution to combine magnetic gradient Inversion and Gravity Gradient Inversion in an information fusion way, where the two kinds of data are complementary to each other and a optimal detection is calculated by the weighted least squares estimation.

This paper is organized as follows. In Section 2, the joint gravity gradient and magnetic gradient inversion algorithms are introduced and analyzed, In Section 3, experiment and stimulation results are discussed. Conclusions are summarized in Section 4.

## 2 Joint Gravity Gradient and Magnetic Gradient Inversion Method

### 2.1 Structure of Joint Geophysical Inversion Method



**Fig. 1.** Block diagram of Magnetic-Gravity Gradient Inversion for Underwater Object Detection

### 2.2 Introduction of Information Fusion Algorithm

In Fig1, after calculating the relative distances from gravity gradient inversion method and magnetic gradient inversion method, which are denoted by  $P_{gg}$  and  $P_{mg}$  respectively, the weighted least squares estimation method in reference [4][5] is constructed to get an optimal relative distance between underwater vehicle and underwater object. The optimal equation is shown as follows:

$$\left\{ \begin{array}{l} \hat{P}_{Joint} = W_{gg} P_{gg} + W_{mg} P_{mg} \\ \delta_T = \sqrt{\sum w_{ig}^2 \delta_{ig}^2} \\ \sum w_{ig} = 1 \\ 0 \leq w_{ig} \leq 1 \\ i = g, m \end{array} \right. \quad (1)$$

Where  $W_{gg}$  and  $W_{mg}$  are weights coming from gravity gradient inversion and magnetic gradient inversion.  $\hat{P}_{Joint}$  is the optimal localization result from two Inversion methods, according to methods introduced in reference [8][9], the relative distance between an underwater object and underwater vehicle can be expressed as follows:

$$\left\{ \begin{array}{l} \frac{\partial B_x}{\partial X} = a = \frac{3\mu x(3r^2 - 5x^2)m_x + y(3r^2 - 5x^2)m_y + z(3r^2 - 5x^2)m_z}{r^7} \\ \frac{\partial B_y}{\partial Y} = b = \frac{3\mu x(3r^2 - 5y^2)m_x + y(3r^2 - 5y^2)m_y + z(3r^2 - 5y^2)m_z}{r^7} \\ c = -(a+b) \\ \frac{\partial B_y}{\partial X} = d = \frac{3\mu y(3r^2 - 5x^2)m_x + x(3r^2 - 5x^2)m_y - 5yzm_z}{r^7} \\ \frac{\partial B_z}{\partial Y} = e = \frac{3\mu z(r^2 - 5y^2)m_y + y(r^2 - 5z^2)m_z - 5yzm_x}{r^7} \\ \frac{\partial B_z}{\partial X} = f = \frac{3\mu z(r^2 - 5x^2)m_x + x(r^2 - 5z^2)m_z - 5yzm_y}{r^7} \\ A_6 = A_5k^5 + A_4k^4 + A_3k^3 + A_2k^2 + A_1k^1 + A_0 = 0 \\ A_0 = d^2(a+2b) - e^2(a-b) + 2def \\ A_5 = -2d[(a-b)(a+2b) + (d^2 + e^2 + f^2)] \\ A_4 = (a-b)^2(a+2b) + d^2(4a-7b) + (f^2 - 2e^2)(a-b) + 6edf \\ A_3 = -4d[(a-b)^2 + (d^2 + e^2 + f^2)] \\ A_2 = (a-b)^2(a+2b) + d^2(4b-7a) + (2f^2 - e^2)(a-b) + 6edf \\ A_1 = 2d[(a-b)(a+2b) - (d^2 + e^2 + f^2)] \\ A_0 = d^2(a+2b) + f^2(a-b) + 2edf \\ q = \frac{[d(k^2-1) - (a-b)k]}{(ek-f)(k+1)} \\ z = \frac{\pm 3}{\sqrt{[(ak+d)q+f]^2 + [(dk+b)q+e]^2 + [(fk+e)q+c]^2}} \\ x = kq \\ y = q \\ P_{mg} = \sqrt{x^2 + y^2 + z^2} \\ P_{gg} = R(x, y, z) = \sqrt{\frac{GM}{\Gamma_x(x, y, z) + \Gamma_y(x, y, z)} \left( 1 - \frac{3}{\left(\frac{\Gamma_x(x, y, z)}{\Gamma_x(x, y, z)}\right)^2 + \left(\frac{\Gamma_y(x, y, z)}{\Gamma_x(x, y, z)}\right)^2 + 1} \right)} \end{array} \right. \quad (2)$$

Where  $\Gamma_{ij}(x, y, z)$  denotes the gravity gradient value with coordinate  $x, y$  and  $z$ , respectively;  $\theta(x, y, z)$  and  $\varphi(x, y, z)$  stand for the relative orientation of an

underwater object;  $R(x, y, z)$  is a relative distance between the underwater vehicle and an underwater object that needs to be detected;  $M$  is the mass of the object.  $G$  is the gravitation constant [4-5]; In order to get the values  $A$ , it is necessary to measurement magnetic gradients (a, b, d, e, f), According to the method in reference [10], seven single-axis magnetometer configuration is as the simplest scheme of measuring magnetic gradient tensor. After obtaining parameters  $A$ , the six order equation in Eq. (2) can be calculated to get the value of  $k$ , then,  $k$  is to calculate  $q$ . Finally, the underwater vehicle position parameters (x, y, z) relative to the underwater object can be calculated by Eq. (2)

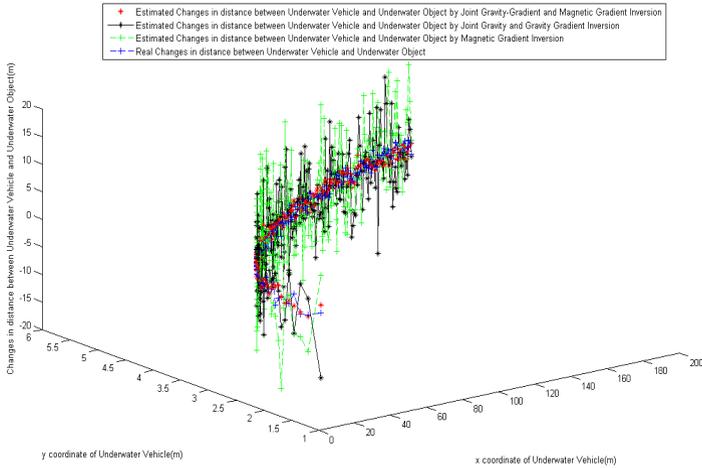
### 3 Stimulation Results

In order to do a comparison with the method in reference [4-5], a cube model was also designed for the estimation of the changes in distance between an underwater vehicle and an underwater object. The relevant parameters are listed in Table 1. Assuming position of underwater vehicle is fixed and underwater object is moving. Based on equation (2), after getting the values of gravity gradient and magnetic gradient tensors, the relative distance between underwater vehicle and underwater object can be calculated. Then equation (1) is to combine each gradient inversion information by weighted least squares estimation method in reference [4][5].

**Table 1.** Parameters of Underwater Object Model

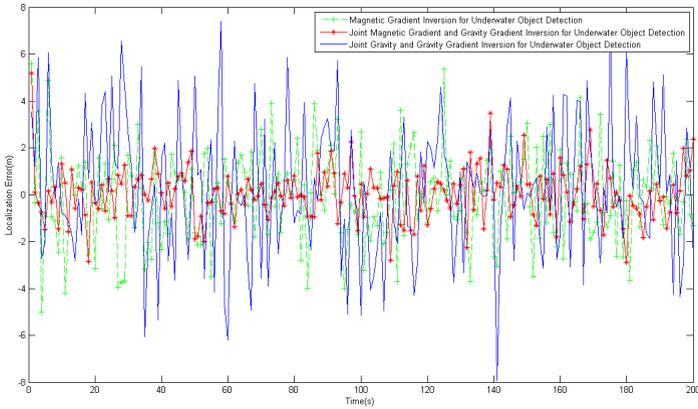
Model	Size (m)
$X$ direction	20
$Y$ direction	20
$Z$ direction	20
Density contrast (kg/m3)	25
Actual mass of object(kg)	$6.624 * 10^4$
$m_x$	$10^6 A \cdot m^2$
$m_y$	$2 \times 10^5 A \cdot m^2$
$m_z$	$10^5 A \cdot m^2$

In this part of the stimulation, a fixed underwater vehicle and a moving underwater object were assumed. The density distribution and the size of the object were ignored and the object was assumed to be a particle. While the object was moving, sequences of varying gravity gradient and magnetic gradient responses at the fixed position of underwater vehicle could be measured simultaneously. According to Eq. (2), relative positions between underwater vehicle and moving object can be calculated. Besides, a constant relative error of 12.33% was added to magnetic gradient inversion method, which means that  $\delta_{mg}$  in (1) was equal to 12.33%. When the relative error of gravity gradient ( $\delta_{gg}$ ) in (1) is 22.65%, here, the weights are calculated with (1) so that  $W_{gg}=0.219$  and  $W_{mg}=0.781$ . The estimation result is shown in Fig. 2.



**Fig. 2.** A Comparison between actual changes in distance between underwater vehicle and underwater object and estimated changes in distance between underwater vehicle and underwater object based on Three Inversion methods

In Fig.2, the actual change in distance between underwater vehicle and underwater object is depicted with a blue line. The estimated change in distance between underwater vehicle and underwater object by the proposed joint gravity gradient and magnetic gradient inversion method is denoted by red stars. The black stars are to denote the estimated changes in distance between underwater vehicle and underwater object by Joint gravity and gravity gradient inversion. Finally, the green line is to illustrate the estimated changes in distance between underwater vehicle and underwater object by magnetic gradient inversion. It is clearly seen that the estimated changes in distance between underwater vehicle and underwater object by joint gravity gradient and magnetic gradient inversion method is in much better agreement with the actual changes in distance between underwater vehicle and underwater object. In fact, the changes in distance between underwater vehicle and underwater object is a good solution to reflect the relative position changes between underwater vehicle and underwater object, the bias in estimated changes in distance between underwater vehicle and underwater object come from measurement outliers from magnetometers and gradiometers. The accuracy of each geophysical sensor has a great impact on the accuracy of underwater object localization. In addition, the fluctuation of various terrains in underwater environment has a great impact on the accuracy of Joint gravity and gravity gradient inversion method. Furthermore, electromagnetic disturbance in underwater environment is another important factor which affects the accuracy of magnetic gradient inversion method. So, the joint gravity gradient and magnetic gradient inversion method is a good solution to eliminate such kinds of negative influences on underwater object detection. The Fig.2 shows its effectiveness in underwater object detection.



**Fig. 3.** Error Comparison among three Underwater Object Detection Methods

In Fig.3, it is clearly to see that Joint Magnetic Gradient and Gravity Gradient Inversion methods for underwater object detection show the best localization performance among three methods. In fact, gravity and gravity gradient is sensitive to the fluctuation of various terrain in the underwater environment, on the contrary, magnetic gradient is less affected by fluctuation of underwater terrain and time-vary disturbances. In this way, magnetic gradient is a good solution to combine with gravity gradient to realize underwater object detection. In the future, it is necessary to pay more attention to improve the accuracy of magnetometer and gradiometer and develop a better solution to realize data fusion method to fuse gravity gradient and magnetic gradient information.

## 4 Conclusions

Underwater Object detection method based on joint gravity gradient and magnetic gradient inversion has been proposed. The weighted least square is introduced to fuse inversion information from gravity gradient inversion and magnetic gradient inversion respectively. With such method, the changes in distance between underwater vehicle and an underwater object can be detected accurately. Simulation results show that the joint inversion method is better than mono-magnetic gradient inversion method and joint gravity and gravity gradient inversion method.

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